PHYS 231 - Sept. 13, 2023

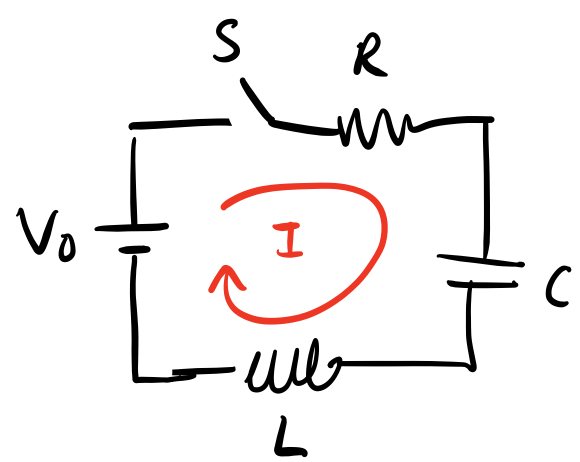
Last Time:

$$|\Delta V_c| = \frac{Q}{C}$$

$$|\Delta V_R| = IR = R\frac{dQ}{dt}$$

$$|\Delta V_L| = L\frac{dI}{dt} = L\frac{d^2Q}{dt^2}$$

LRC Circuit



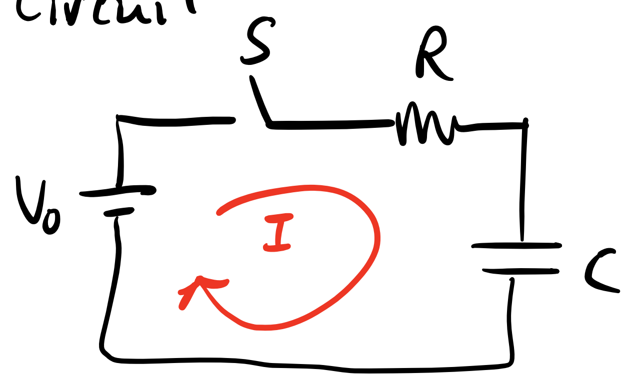
$$V_0 = \frac{1}{C}Q + R\frac{dQ}{dt} + L\frac{d^2Q}{dt^2}$$

2nd order differential eq.

Can solve for time dependence of Q.

Will look at this sol'n later in the course.

RC Circuit



Assume capacitor uncharged at $t=0$ when switch is closed.

Initial Condition: $Q(t=0)=0$.

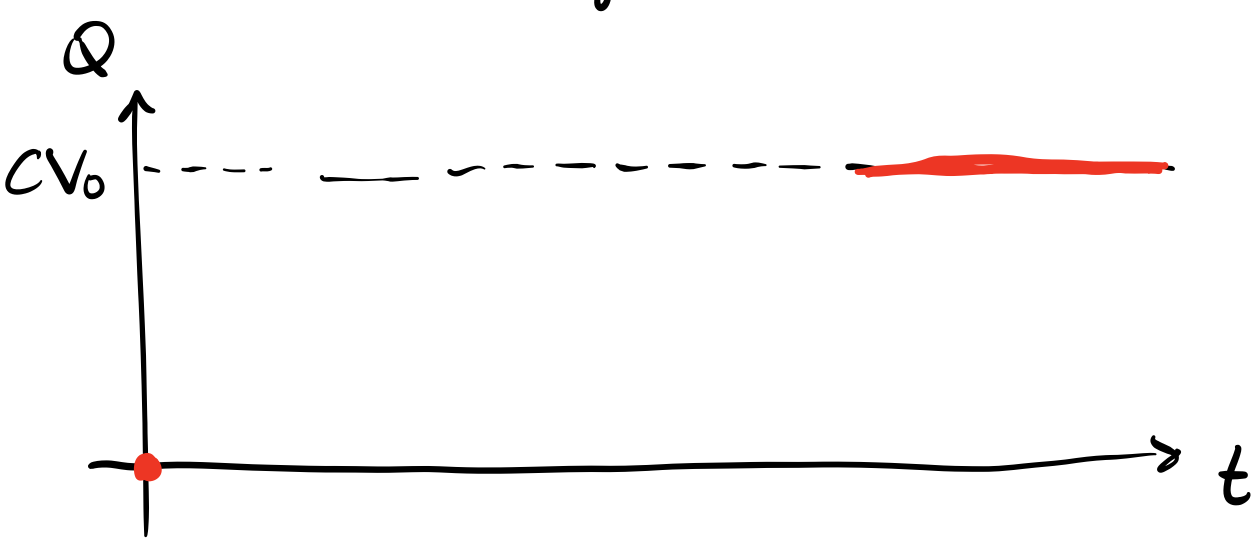
$$V_0 = \frac{1}{C}Q + R\frac{dQ}{dt}$$

First order differential eq.

$$\Rightarrow CV_0 = Q + RC\frac{dQ}{dt}$$

A long time after switch is closed $(t\to\infty)$ reach a new equilibrium $\left(\frac{dQ}{dt}=0\right)$

When this happens, get $Q_0 = CV_0$ @ $t\to\infty$.



Express $Q(t)$ as sum of some time-dep. $Q_h(t)$ and a constant Q_0 $(t \to \infty)$.

$$Q(t) = Q_h(t) + \underbrace{Q_0}_{CV_0} \quad ①$$

Sub ① into our first-order diff. eq'n.

$$\cancel{CV_0} = \left(Q_h(t) + \cancel{CV_0}\right) + RC \frac{d}{dt}\left(Q_h(t) + CV_0\right)$$

called "homogeneous" sol'n.

$$\therefore 0 = Q_h(t) + RC \frac{dQ_h(t)}{dt}$$

$$Q_h(t) = -RC \frac{dQ_h(t)}{dt} \quad \text{(mult. by } dt\text{)}$$

$$Q_h(t)\, dt = -RC\, dQ_h(t) \quad \text{(divide by } -RCQ_h(t)\text{)}$$

$$-\frac{1}{RC}\, dt = \frac{dQ_h(t)}{Q_h(t)} \quad \text{(integrate)}$$

define $RC = \tau$ (time constant).

$$-\frac{1}{\tau}\int dt = \int \frac{dQ_h(t)}{Q_h(t)}$$

$$-\frac{1}{\tau}t + \underset{\substack{\uparrow \\ \text{integration const.}}}{B} = \ln Q_h(t)$$

$$\therefore\ Q_h(t) = e^{(-t/\tau + B)}$$
$$= e^{-t/\tau}\underbrace{e^{B}}_{\equiv B'}$$

$$\therefore\ Q_h(t) = B' e^{-t/\tau}$$

Since $Q(t) = Q_h(t) + CV_0$

$$= B' e^{-t/\tau} + CV_0.$$

Still need to find B'.

Using initial condition $Q(t=0)=0$ to find B'.

$$Q(t=0) = 0 = B' + CV_0$$

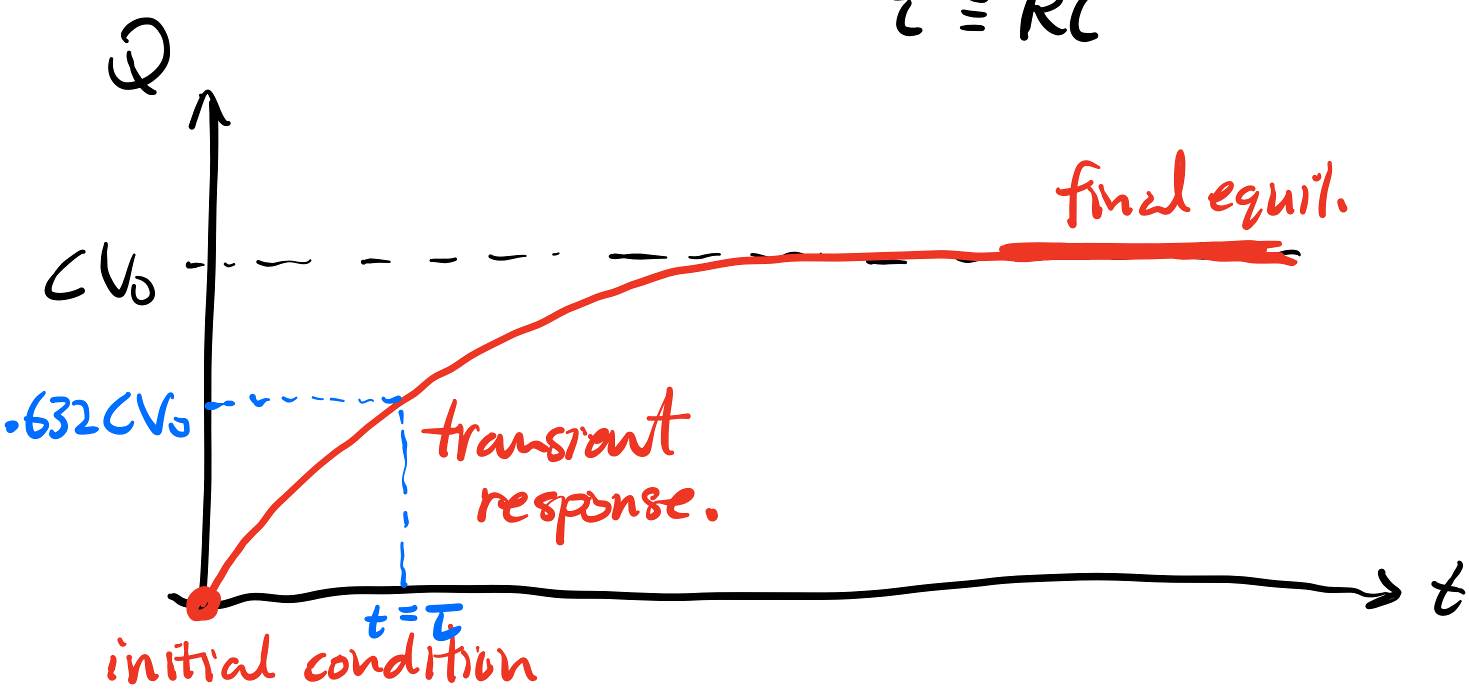
$$\therefore \quad B' = -CV_0$$

Final sol'n for $Q(t)$ is:

$$Q(t) = -CV_0 e^{-t/\tau} + CV_0$$

$$\boxed{Q(t) = CV_0\left[1 - e^{-t/\tau}\right]}$$

$$\tau \equiv RC$$



Find the charge on capacitor when
$t = \tau$

$$Q(t=\tau) = CV_0 \underbrace{\left[1 - e^{-1}\right]}_{0.632}$$

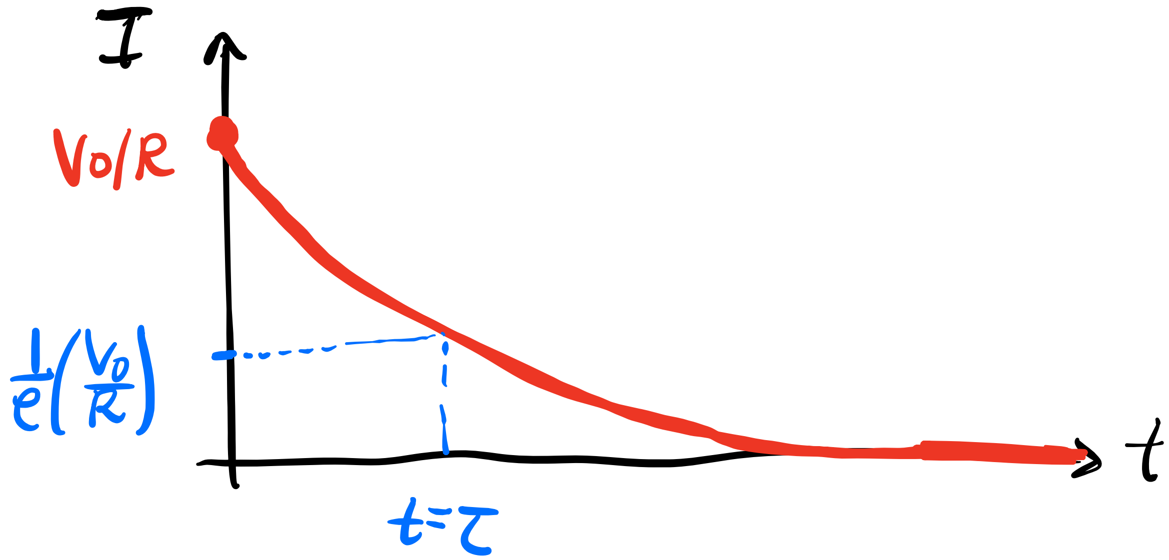
To find the current, take time derivative of charge.

$$I = \frac{dQ}{dt} = \frac{d}{dt}\left(CV_0\left[1 - e^{-t/\tau}\right]\right)$$

$$= -CV_0 \frac{d}{dt} e^{-t/\tau}$$

$$= \frac{CV_0}{\tau} e^{-t/\tau}$$

$$\boxed{I = \frac{V_0}{R} e^{-t/\tau}}$$



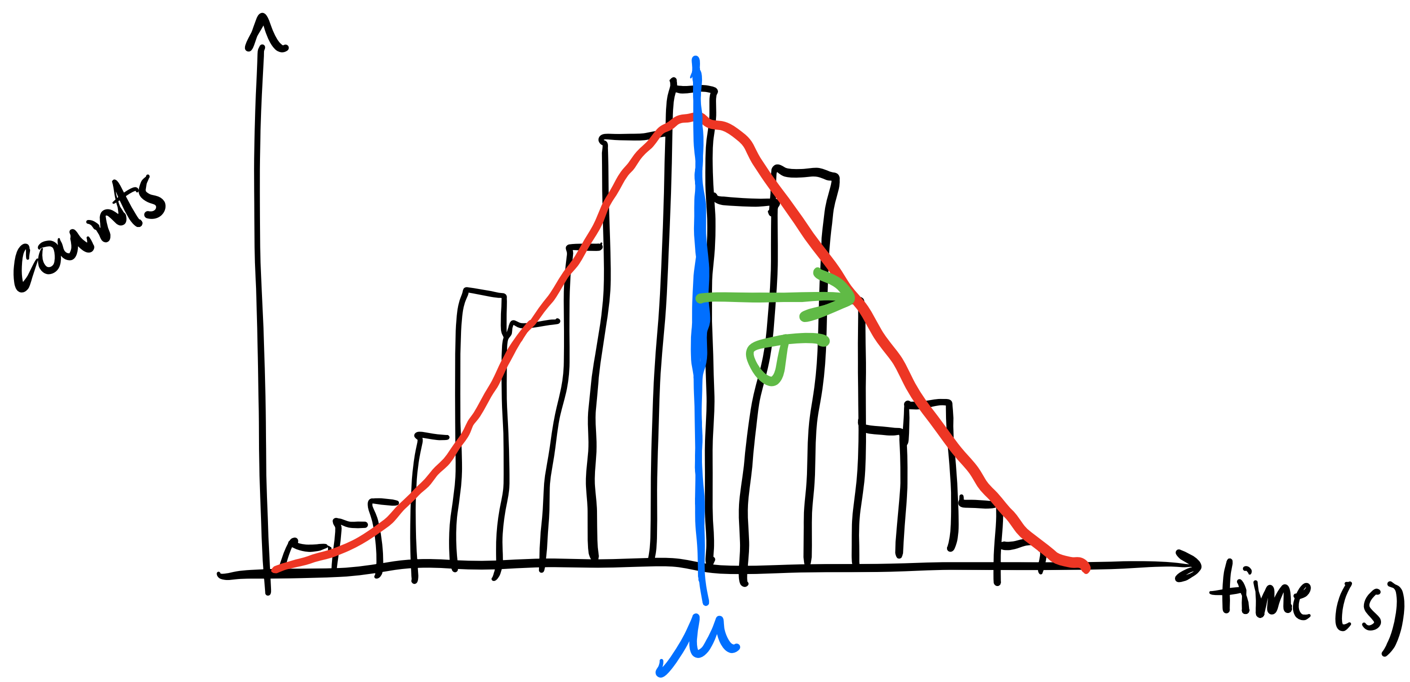
Uncertainties & Propagation of Errors.

Every quantity that is measured has an associated uncertainty.

Experiment: Drop a penny from a tall building & meas time for it to reach the gnd. For each trial we will meas. a unique fall time.

trial #	time (s)
1	148.25
2	151.1
3	150.4
⋮	⋮
N	149.7

Plot the measured data as a histogram



If the uncertainty in the meas. is random, then the data will follow a Gaussian or normal dist'n (Bell curve).

Can characterize the shape of the dist'n using the mean or average μ.

$$\mu = \frac{1}{N} \sum_{i=1}^{N} t_i$$

and the standard deviation, σ.

σ characterizes the width of the dist'n.

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (t_i - \mu)^2}$$